Conclusions.—1. The stars of class B show systematic differences in the proper motions in the two regions of the Milky Way at right angles to the direction of solar motion.

2. The solar velocities derived from the B stars in the northern and southern hemispheres separately differ, that from the northern stars being the smaller.

3. These conditions appear to be best explained by a general motion of rotation of the system of stars in a retrograde direction about an axis perpendicular to the Milky Way.

The details of these preliminary investigations will be published elsewhere.

THEORY OF AN AEROPLANE ENCOUNTERING GUSTS

By Edwin Bidwell Wilson

DEPARTMENT OF MATHEMATICS, MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Received by the Academy, April 10, 1916

A number of references to the theory of the stability of aeroplanes may be given,[1] but on their behavior in gusts little has been printed,[2] though much is probably known and held secret by foreign governments. Last summer I was asked to develop a theory of the effect of gusts upon (symmetric) aeroplanes, in particular upon a definite machine of which the aerodynamical coefficients were furnished me by Mr. Hunsaker. Two questions were to be answered: First, what is the effect of a gust upon the longitudinal motion in the uncontrolled machine; second, what is the effect when the machine is so constrained by an automatic device as to maintain its axis horizontal? My report, submitted to our National Advisory Committee for Aeronautics, will appear as a part of the *Report of the National Advisory Committee for Aeronautics, 1915* (Senate Document No. 268, 64th Congress, 1st Session). By the courtesy of the Committee I am permitted to give here a brief account of this work.

In treating the motion of an aeroplane the machine is referred to a set of moving axes x, y, z fixed in the body, and directed backward, to the left, and upward in normal horizontal flight with velocity $-U$. Departures from this standard condition of flight are treated by the method of small oscillations. If $U + u$, v, w, p, q, r be the linear and angular velocities in the disturbed motion and if u_1, v_1, w_1, p_1, q_1, r_1 be the linear and rotational velocities of the gusts (taken with the proper sign), the aerodynamic forces acting upon the machine will vary from those acting in the standard conditions by amounts which may be determined by the eighteen aerodynamic constants X_u, X_w, X_q, Z_u, Z_w, Z_q, M_u, M_w, M_q,

Y_v, Y_p, Y_r, L_v, L_p, L_r, N_v, N_p, N_r, of which the last nine refer to the 'lateral' motion which was not discussed.

The equations for the 'longitudinal' motion of the free machine are

$$\begin{aligned} du/dt - g\theta - X_u u - X_w w - X_q q &= X_u u_1 + X_w w_1 + X_q q_1, \\ dw/dt - Uq - Z_u u - Z_w w - Z_q q &= Z_u u_1 + Z_w w_1 + Z_q q_1, \\ B/M.\, dq/dt - M_u u - M_w w - M_q q &= M_u u_1 + M_w w_1 + M_q q_1, \end{aligned} \tag{1}$$

where θ is the inclination of the x-axis, and $B/M = k^2$ is the square of the radius of gyration about the y-axis. For small oscillations these equations are linear with constant coefficients. The solution of (1) may be carried out by the ordinary method. In particular the solution may be cast in such form that a number of different gusts may be discussed very rapidly after a certain amount of preliminary calculation has been made.

The data for the given machine are as follows:

$$\begin{array}{lll} X_u = -0.128, & X_w = 0.162, & X_q = Z_q = M_u = 0, \\ Z_u = -0.557, & Z_w = -3.95, & M_w = 1.74, \\ M_q = -150, & k^2 = 34, & U = -115.5. \end{array}$$

The numerical equation for u is

$$\begin{aligned} &(D^4 + 8.49\, D^3 + 24.5\, D^2 + 3.385\, D + 0.917)u = \\ &\quad -(0.128\, D^3 + 1.16\, D^2 + 3.385\, D + 0.917)u_1 \\ &\quad + (0.162\, D^2 + 0.715\, D + 1.647)\, Dw_1 - (59.37\, D + 560.6)\, q_1, \end{aligned}$$

where $D = d/dt$, with similar results for w and θ. The roots of $D^4 + 8.49\, D^3 + 24.5\, D^2 + 3.385\, D + 0.917 = 0$ are $-4.18 \pm 2.43i$ and $-.0654 \pm .187i$. The solutions are therefore of the form $u = e^{-4.18t}$ $(A \cos 2.43t + B \sin 2.43t) + e^{-.0654t}\, (C \cos .187\, t + D \sin .187t) + I_u$, where A, B, C, D are constants of integration and I_u is a particular integral obtained from some particular gust u_1, w_1, q_1. There are similar equations for w and θ. Of the twelve constants of integration only four are independent, and the relations between them are independent of the particular integrals I (provided the gust is not tuned to the free oscillation). It is therefore possible to set down, once for all, formulas for the coefficients, A, B, C, D in terms of the initial values of the particular integrals I_u, I_w, I_θ, I'_θ. Where the machine is in normal flight with $u = w = \theta = q = 0$.

The type of gust chosen was $J\ (1 - e^{-rt})$, where J is an intensity-factor. This gust rises from 0 to J in an infinite time, but the greater part of the rise occurs in the time $1/r$ or $2/r$. If we set u_1, w_1, or q_1 equal to $J\ (1 - e^{-rt})$, we obtain a head-gust, up-gust, or rotary gust. The

values 0.2, 1.0, 5.0 were successively assigned to r to correspond to mild, moderate, and sharp gusts.

The Uncontrolled Machine.—The result for a head-gust is that, during 15 to 18 seconds after striking the gust, the machine soars aloft to a height from $4J$ to $4\frac{1}{2}J$ above its initial level and subsequently executes a damped oscillation about the level $3\frac{1}{2}J$ above the original. The effect is not seriously affected by the sharpness of the gust (measured by r) but is proportional to its intensity (measured by J). The shock of the gust is small. For a rear-gust the effects are reversed in sign, the machine falls. A severe rear-gust coming upon a machine flying low might therefore produce serious consequences.

The major effect of an up-gust is to carry the aeroplane bodily upward with it. The oscillations are small, but in the case of a sharp gust, the initial acceleration may be considerable and 'bump' the pilot. The effects for down-gusts are reversed in sign.

Little appears to be known in regard to the extent or intensity or permanence of rotary aerial motions, and in the absence of such meteorological information, no satisfactory practical conclusions may be stated as to the effect of these gusts upon flight. The analysis, however, indicates that rotary gusts of relatively small intensity may have relatively large effects on the motion.

The Constrained Machine.—If the machine is so constrained that $\theta = 0$, the equations of motion reduce to the first two of (1). Substitution of the numerical data shows that the motion, when disturbed, is no longer oscillatory, but approaches the original condition asymptotically. The riding of the machine should therefore be steadier. The use of data obtained for lower speeds than $U = -115.5$ feet per second shows, however, that the constrained aeroplane loses its dynamical stability at a higher speed than the free machine, and to that extent is more dangerous in landing.

In a head-gust the constrained machine soars steadily (without oscillation) up to some J feet above its initial level. Thus the upward swoop of $4J$ to $4\frac{1}{2}J$ has been eliminated and the final increase in level is less than one-third as much as before. In a down-gust the effects are reversed in sign.

The up-gust is again important only for its general convective effect upon the machine. In case the gust is sharp, there may, however, be a considerable momentary acceleration or 'bump' at first.

It is interesting to note that if an aeroplane, equipped with the automatic device, is riding on a breeze which has a leisurely periodic gustiness, a habile pilot may 'suck' considerable energy out of the gusts. For

he may ride up in the head-gust, with the constraining device thrown off, to a height of some $4J$ and then, with the device working, need drop only about J in the rear-gust. In this manner he can gain the amount $3J$ in altitude during each period. To avoid interference the successive maxima of the gusts should be at least 30 seconds apart for this machine.

[1] G. H. Bryan, *Stability in Aviation*, Macmillan, 1910; L. Bairstow, *Technical Report of the Advisory Committee for Aeronautics*, London, 1912–13; J. C. Hunsaker, these PROCEEDINGS, 2, 278 (1916).

[2] A general lecture by Glazebrook, *Aeronautical Journal*, 272-301, July, 1914, should be cited.

TERMS OF RELATIONSHIP AND SOCIAL ORGANIZATION

By Truman Michelson

BUREAU OF AMERICAN ETHNOLOGY, WASHINGTON

Received by the Academy, April 17, 1916

Some years ago Kroeber undertook to show that terms of relationship are linguistic and psychological phenomena.[1] Recently Rivers has attempted to overthrow this view completely, and holds that they are sociological phenomena, and consequently that it is entirely possible to infer marriage customs and social organization from these terms.[2] Lowie lately has to a certain extent followed Rivers; he has not followed the latter's survival-theories,[3] and it is doubtful if many American ethnologists will do so.[4]

In this paper I wish to develop Kroeber's thesis from a different angle, and also to make a point on my own account. Long ago Morgan saw that for the most part the terms of relationships are identical in all Algonquian languages with phonetic changes, and consequently for the greater part must go back to the original parent language.[5] Now insofar as this is the case, to this extent terms of relationship are linguistic phenomena. For example, the Fox are organized in exogamic gentes with descent in the male line, and the Plains Cree have no gentile organization at all, yet have at least seventeen terms of relationship in common with the Fox. Again the Delaware who are organized in exogamic clans with descent in the female line have some terms of relationship in common with both the Fox and Cree. Similarly the Shawnee who have an entirely different organization from any of the above mentioned Algonquian tribes,[6] nevertheless have many terms in common with Plains Cree and Fox, and some with Delaware. Accordingly it is obvious that social organization is not the sole factor in terms of relationship. It may be objected that though Plains Cree and Fox possess